CONVECTIVE HEAT TRANSFER FROM A SUDDENLY-DEVELOPED HORIZONTAL HOT SPOT

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This article reports results of an experimental study of nonsteady heat transfer in the atmosphere from a suddenly-heated thin plate lying on a horizontal nonheat-conducting base. The study augments the results reported in [1]. Problems of nonsteady processes connected with the convective cooling of suddenly-heated objects were discussed in [2-5].

In accordance with [1], we will examine the problem in the following formulation. Let a circular hot spot be suddenly formed at the initial moment of time $t = 0$ in the gravitational field g on a horizontal nonheat-conducting surface S (by suddenly, we mean that the spot is formed over a period of time much shorter than the characteristic time of formation of motion). The hot spot has an axisymmetric temperature distribution over the surface $T_{S1}(r) = T_0 + T_S(r)$ [T_0 is the temperature of the external medium, while r is the distance from the center of the spot (we will henceforth use cylindrical coordinate system (r, z))]. We will assume that the heated layer on the nonconducting surface is infinitely thin but has a finite surface heat capacity C_S.

The half-space above the plane S is filled by a medium characterized by the density ρ, pressure p, and volumetric heat capacity C_p. All of these quantities are functions of the state of the medium and may therefore change during motion both over time and from point to point. At the initial moment of time in the region of space not disturbed by the effect of the hot spot, $\rho = \rho_0$, $p = p_0$, $T = T_0$.

It was shown in [1] that the temperature distribution over the surface of a hot spot at $t > 0$ can be represented in the form

$$T^0 = T/T_{S0} = T^0(r^0, z^0, t^0),\ z^0 = z/\Lambda_0,\ r^0 = r/\Lambda_0,\ t^0 = t/\lambda_0 \tag{1}$$

[T_{S0} is the characteristic (or maximum) value of the temperature gradient on the surface S at $t = 0$]. The parameters Λ_0 and λ_0, having the dimensions of length and time, respectively, can be regarded as the characteristic dimension of the perturbed region and the time of its formation. In [1], the methods of dimensional theory were used to determine the parameters Λ_0 and λ_0 to within the multiplier Λ:

$$\Lambda_0 = \Lambda\sqrt{Qg/(C_pT_0^2\rho_0)},\quad \lambda_0 = \sqrt{\Lambda_0 T_0/(T_{S0}g)}. \tag{2}$$

Here and below, H_0 is the characteristic dimension of the hot spot; Q is the amount of energy transferred to the atmosphere by the hot spot.

It was assumed in [1] that Λ and C_S are constants, and it was shown that the assumption that Λ is constant (with C_S = const) within a certain range of the parameters which determine motion is supported by observations. It was further shown in [1] that there are limits beyond which experimental data analyzed in accordance with (1) deviates from the single curves plotted in the coordinates (1) with Λ = const and C_S = const.

If we do not assume that Λ and C_S are constant, then when we use the series of governing parameters F, i_0, T_{S0}, gT_{S0}/T_0, and C_S [$F = gQ/(C_pT_0)$ is the weight deficit and i_0 is the enthalpy of the medium] we obtain the unique dimensionless combination $\eta = \sqrt{Fi_0}/(C_ST_{S0})$. It follows from this that, in the general case, $\Lambda = \Lambda(\eta)$. Since $Q \sim C_ST_{S0}H_0^2$ (see [1]), it follows from the relations found already that

$$\eta = H_0\sqrt{\rho_0 g/(C_ST_{S0})}. \tag{3}$$

At $\eta \ll 1$, we can expand $\Lambda(\eta)$ into a series in η: $\Lambda(\eta) = \Lambda(0) + \eta\Lambda'(0) + \ldots$ Given sufficiently small values of the parameter η, $\Lambda = \Lambda(0)$ = const. This case actually exists

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at C_s = const and was examined in [1]. Here, with the use of (1) and (2), the coordinates, time, and the temperature gradient (for example) for the initial temperature distribution on the surface of the hot spot

$$T_s(r) = T_{s0}\left(1 + r^2/H_0^2\right)^{-3/2} \tag{4}$$

can be reduced to dimensionless form

$$\Lambda_0 = \sqrt{\frac{\mu_a T_{s0}\rho_a T_{0a} H_0^2 C_s^0}{T_0^2 \rho_0 C^2}}, \quad \lambda_0 = \sqrt[4]{\frac{\mu_a C_s^0 T_0^2 \rho_a H_0^2}{T_{s0}\rho_0 g^2 C^2 T_{0a}}},$$

$$z^0 = \frac{zC}{H_0}\sqrt{\frac{T_0^2\rho_0}{T_{s0}T_{0a}\rho_a C_s^0}}, \quad r^0 = \frac{rC}{H_0}\sqrt{\frac{T_0^2\rho_0}{T_{s0}T_{0a}\rho_a C_s^0}}, \quad t^0 = t\sqrt[4]{\frac{T_{s0}T_{0a}\rho_0 g^2 C^2}{T_0^2\rho_a H_0^2 C_s^0}}, \tag{5}$$

$$T^0 = \frac{T}{T_{s0}}, \quad \mu_a = \frac{2\pi\Lambda^2 C_{s0} g}{C_{pa}^2 T_{0a}\rho_a}, \quad C = \frac{C_p}{C_{pa}}, \quad C_s^0 = \frac{C_s}{C_{s0}},$$

where C_{pa} and ρ_a are the heat capacity and density of air at a pressure of 10^5 Pa and a temperature T_{0a} = 290 K, C_{s0} = 800 J/(m^2·K).

For the initial temperature distribution on the surface of the hot spot

$$T_s(r) = T_{s0} \text{ at } r \leqslant r_0, \quad T_s(r) = 0 \text{ at } r > r_0$$

Eq. (5) has the same form. However, here the parameter H_0 is replaced by r_0. Equations (5) differ from the corresponding relations obtained in [1] in the fact that C_s is introduced into the former in explicit form (in [1], this parameter was included in the expression for μ_a).

The transformations of (5) express general laws governing the development of the motions being examined here. In order to obtain formulas to determine such quantities as the characteristic time of development of the motion and the dimensions of the atmospheric region disturbed by the effect of the hot spot, as well as to calculate the heat-transfer coefficient k, we will examine the problem of the cooling of the spot. The dynamics of cooling can be established on the basis of Newton's law [6]. Here, as is known, the transfer of heat to a unit area of the atmosphere S is proportional to T_s – the temperature of the hot spot:

$$dQ_s/dt = C_s dT_s/dt = -kT_s. \tag{6}$$

It follows from (6) that

$$T_s = T_{s0}e^{-kt/C_s}. \tag{7}$$

The value of C_s does not change during experiments. The coefficient k depends on the character of the heat-transfer process and on the density and temperature of the environment. It follows from (5) and (7) that

$$k = k(\eta) = -(C_s/t)\ln(T_s/T_{s0})$$

or

$$k^0(\eta) = \frac{C_s^0}{t^0}\ln T_s^0, \tag{8}$$

where $k^0 = k\lambda_0/C_{s0}$. Here,

$$k = k^0 C_{s0}\sqrt[4]{\frac{T_{s0}\rho_0 g^2 C^2 T_{0a}}{T_0^2\rho_a H_0^2 C_s^0}}.$$

It should be noted that the dimensionless heat-transfer coefficient is a function of the parameter η:

$$k^0 = k^0(\eta). \tag{9}$$

At $\eta \ll 1$, it can be represented in the form

$$k^0(\eta) = k^0(0) + \eta(k^0)'(0) + \ldots \tag{10}$$

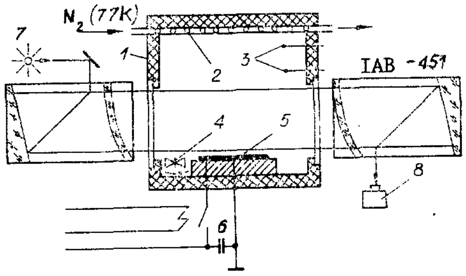

Fig. 1

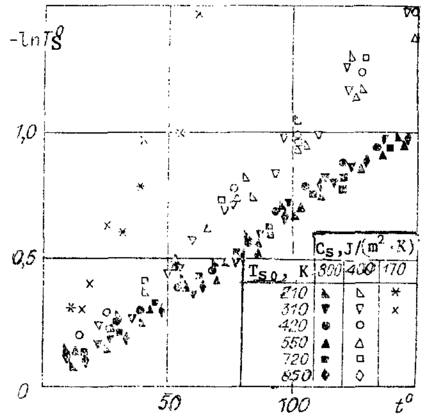

Fig. 2

In the case of sufficiently small η, we obtain $k^0 \simeq k^0(0)$. Equations (7) and (8) show that the coefficient k can be found by means of the cooling curve of the hot spot. Equation (10) makes it possible to approximately obtain the error permitted with the replacement of k^0 by $k^0(0)$, i.e., to determine $k^0 - k^0(0) = \delta(\eta)$.

To check the above theory and to find k, we conducted experiments on two units. The design of the first unit and the experimental method used in this case were discussed in [1]. The tests were conducted in an altitude chamber that ensured the possibility of variation of the initial density and pressure of the medium. The device used to create the hot spot was placed inside the chamber. The convective flow was visualized with an IAB-451 Teploer unit. The temperature of the heated gas was measured with thermometers made by the DISA company, while the change in the temperature of the hot spot was measured with thermocouples.

The second unit was designed to study the effect of the initial temperature of the atmosphere on the laws governing the formation and rise of the convective flow. A sketch of the unit is shown in Fig. 1. The tests were conducted in the 0.4 × 0.4 × 0.4 m chamber 1, which had thermally insulated walls. A radiator 2, serving as a conduit for pumped liquid nitrogen, was placed in the top part of the chamber to cool the gas. Here, air inside the chamber was cooled to a prescribed temperature determined by the rate of pumping of the nitrogen. The temperature of the gas in the chamber was monitored with thermocouples 3 located at several levels. A fan 4 which moved the gas, and thus increased heat transfer from the radiator, prevented stratification in the chamber. The fan was turned off immediately prior to the creation of the hot spot. To obtain air heated above room temperature in the chamber, a heater was located above the fan.

The hot spot was created (as in the first unit) on the surface of a nonheat-conducting plate with a heater 5 located on the plate. The foil heater had a spiral opening through which we discharged a high-voltage capacitor bank 6 at the initial moment of time. The initial temperature distribution on the surface of the hot spot corresponded to Eq. (4). We used a capacitor bank with a total capacitance of 2400 μF charged up to 0.5-1.5 kV. This ensured that the temperature at the center of the hot spot varied within the range 100-800 K. The parameter C_S was varied by replacing the heater by a similar heater made of foil of different thickness. In Fig. 1, 7 is the light source and 8 is the film camera.

Figure 2 uses the dimensionless coordinates (5) to show data on the change in the hot spot in relation to time with different initial hot-spot temperatures, surface heat capacities, and initial ambient temperatures. It is evident that, except for the initial section – where the inertia of the thermocouples plays a role – the experimental points are grouped along single curves. Figure 3 shows the relation $k^0 = k^0(\eta)$.

Again using the dimensionless coordinates (5), Fig. 4 shows graphs of the motion $z^0 = z^0(t^0)$ of the crest of the temperature wave for different values of T_{S0} and T_0. The solid line was taken from [1] and corresponds to the averaged curve of motion of the temperature wave studied with different values of T_{S0}, H_0, and C_p. The data we obtained with variation of the initial temperature of the medium is grouped around this line with a small degree of scatter.

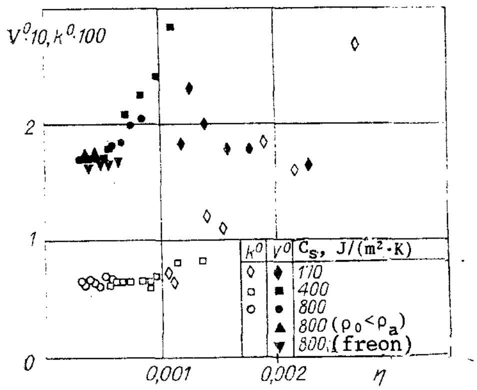

Fig. 3

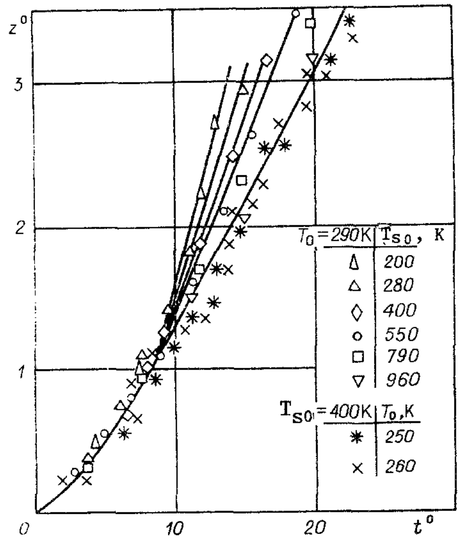

Fig. 4

Figure 4 shows that at the point with the coordinates $z^0 = 1$, $t^0 = 9$, the curves $z^0 = z^0(t^0)$ for $T_{s0} < T_k$ $[T_k = T_k(C_s)]$ deviate from the main curve (the experimental points are located along straight lines whose slope depends on T_{s0}). As a measure of the deviation, we took the rate of rise of the top point of the thermal wave $V^0 = dz^0/dt^0$. Graphs of the relation $V^0 = V^0(\eta)$ are also shown in Fig. 3, from which it is evident that the relations $\Lambda \simeq \Lambda(0)$ and $k^0 \simeq k^0(0)$ are approximately valid at $\eta < 10^{-3}$. With large values of η, both Λ and k^0 depend appreciably on η. It should be noted that the data obtained make it possible to determine $k^0(0) = 6 \cdot 10^{-3}$. The heat-transfer coefficient can be calculated from Eq. (8).

Thus, the similarity observed in [1] for the motions of an atmosphere disturbed by a suddenly-developing high-temperature hot spot does indeed take place at $\eta < 10^{-3}$ (assuming similarity of the initial temperature distribution as well). When $\eta > 10^{-3}$, the condition η = idem must be satisfied for similarity to be maintained.

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